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AN EPITOME OF NAVIGATION

ORIGINALLY BY

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Example 2.—A ship at lat. 75°31′.7 N, long. 79°08′.7 W, in Baffin Bay, steams 263.5 miles on course 155°.

Required.—(1) Latitude and (2) longitude of point of arrival.

Solution.—By computation:

D	263.5 mi.	log	2.42078		
C	155°	l cos	9.95728	l tan	9.66867
l	238′.8 S	log	2.37806		
m	846.3			log	2.92752
DLo	394′.6 E			log	2.59619

L_1	75°31′.7 N	M_1	7072.4	λ_1	79°08′.7 W
l	3°58′.8 S			DLo	6°34′.6 E
(1) L_2	71°32′.9 N	M_2	6226.1	(2) λ_2	72°34′.1 W
		m	846.3		

By traverse table:

D	l	m	DLo
200.0	181.3	800.0	373.0
60.0	54.4	40.0	18.6
3.0	2.7	6.0	2.8
0.5	0.5	0.6	0.3
263.5	238.9	846.6	394.7

L_1	75°31′.7 N	M_1	7072.4	λ_1	79°08′.7 W
l	3°58′.9 S			DLo	6°34′.7 E
(1) L_2	71°32′.8 N	M_2	6225.8	(2) λ_2	72°34′.0 W
		m	846.6		

The labels (N, S, E, W) of l, DLo, and C are determined by noting the direction of motion or the relative positions of the two places.

If the course is near 090° or 270°, a small error in C introduces a large error in DLo. The solution for C to the nearest 0°.1 only, as by traverse table, may introduce a large error in distance if the course is near 090° or 270°.

818. Rhumb lines and great circles.—The principal advantage of a **rhumb line** is that it maintains constant true direction. A ship following the rhumb line between two places does not change true course. A rhumb line makes the same angle with all meridians it crosses and appears as a straight line on a Mercator chart. It is adequate for most purposes of navigation, bearing lines (except long ones, as those obtained by radio) and course lines both being plotted on a Mercator chart as rhumb lines, except in high latitudes. The equator and the meridians are great circles, but may be considered special cases of the rhumb line. For any other case, the difference between the rhumb line and the great circle connecting two points increases (1) as the latitude increases, (2) as the difference of latitude between the two points decreases, and (3) as the difference of longitude increases. It becomes very great for two places widely separated on the same parallel of latitude far from the equator.

A **great circle** is the intersection of the surface of a sphere and a plane through the center of the sphere. It is the largest circle that can be drawn on the surface of the sphere, and is the shortest distance, along the surface, between any two points on the sphere. Any two points are connected by only one great circle unless the points are antipodal (180° apart on the earth), and then an infinite number of great circles passes through them. Thus, two points on the same meridian are not joined by any great circle other than the meridian, unless the two points are antipodal. If they are the poles, *all* meridians pass through them. Every great circle bisects every other great circle. Thus, except for the equator, every great circle lies half in the northern hemisphere and half in the southern hemisphere. Any two points 180° apart on a great circle have the same latitude numerically, but contrary names, and are 180° apart in longitude. The point of greatest latitude is called the **vertex.** For each great circle there is one of these in each hemisphere, 180° apart. At these points the great circle is tangent to a parallel of latitude, and hence its direction is due east-west. On each side of these vertices the direction changes progressively until the intersection with the equator is reached, 90° away, where the great circle crosses the equator at an angle equal to the latitude of the vertex. As the great circle crosses the equator, its change in direction reverses, again approaching east-west, which it reaches at the next vertex.

On a Mercator chart a great circle appears as a sine curve extending equal distances each side of the equator. The rhumb line connecting any two points of the great circle on the same side of the equator is a chord of the curve, being a straight line nearer the equator than the great circle. Along any intersecting meridian the great circle crosses at a higher latitude than the rhumb line. If the two points are on opposite sides of the equator, the direction of curvature of the great circle relative to the rhumb line changes at the equator. The rhumb line and great circle may intersect each other, and if the points are equal distances on each side of the equator, the intersection takes place at the equator.

819. Great-circle sailing is used when it is desired to take advantage of the shorter distance along the great circle between two points, rather than to follow the longer rhumb line. The arc of the great circle between the points is called the **great-circle track.** If it could be followed exactly, the destination would be dead ahead throughout the voyage (assuming course and heading were the same). The rhumb line *appears* the more direct route on a Mercator chart because of chart distortion. The great circle crosses meridians at higher latitudes, where the distance between them is less.

The decision as to whether or not to use great-circle sailing depends upon the conditions. The saving in distance should be worth the additional effort, and of course the great circle should not cross land, or carry the vessel into dangerous waters or excessively high latitudes. A slight departure from the great circle or a modification called composite sailing (art. 825) may effect a considerable saving over the rhumb line track without leading the vessel into danger. If a fix indicates the vessel is a considerable distance to one side of the great circle, the more desirable practice often is to determine a new great-circle track, rather than to return to the original one.

Since a great circle is continuously changing direction as one proceeds along it, no attempt is customarily made to follow it exactly, except in polar regions (ch. XXV). Rather, a number of points are selected along the great circle, and rhumb lines are followed from point to point, taking advantage of the fact that for short distances a great circle and a rhumb line almost coincide.

The number of points to use is a matter of personal preference, a large number of points providing closer approximation to the great circle but requiring more frequent change of course. As a general rule, each 5° of longitude is a convenient length. Legs of equal length are not provided in this way, but this is not objectionable under normal conditions.

If a magnetic compass is used, the variation for the middle of the leg is usually used for the entire leg. In some areas the change in variation and the change in course due to convergence of the meridians are in opposite directions and of about the same magnitude. In these areas the same magnetic course can be used for relatively long distances. The change of deviation with change of heading may also be a consideration.

The problems of great-circle sailing can be solved by (1) chart (art. 820), (2) conversion angle (art. 821), (3) computation (art. 822), (4) table (art. 823), (5) graphically, or (6) mechanically. Of these, (5) and (6) are but graphical or mechanical solutions of (3). They usually provide solution only for initial course and the distance, and are not in common use.

820. Great-circle sailing by chart.—Problems of great-circle sailing, like those of rhumb line sailing, are most easily solved by plotting directly on a chart. For this purpose the U. S. Navy Hydrographic Office publishes a number of charts on the gnomonic projection (art. 317), covering the principal navigable waters of the world. On this projection any straight line is a great circle, but since the chart is not conformal (art. 302), directions and distances cannot be measured directly, as on a Mercator chart. An indirect method is explained on each chart.

The usual method of using a gnomonic chart is to plot the great circle and, if it provides a satisfactory track, to determine a number of points along the track, using the latitude and longitude scales in the immediate vicinity of each point. These points are then transferred to a Mercator chart or plotting sheet and used as a succession of destinations to be reached by rhumb lines. The course and distance for each leg is determined by measurement on the Mercator chart or plotting sheet. This method is illustrated in figure 820, which shows a great circle plotted as a straight line on a gnomonic chart and a series of points transferred to a Mercator chart. The arrows represent corresponding points on the two charts. The points can be plotted directly on plotting sheets without the use of a small-scale chart, but the use of the chart provides a visual check to avoid large errors, and a visual indication of the suitability of the track.

Since gnomonic charts are normally used only because of their great-circle properties, they are often popularly called **great-circle charts.**

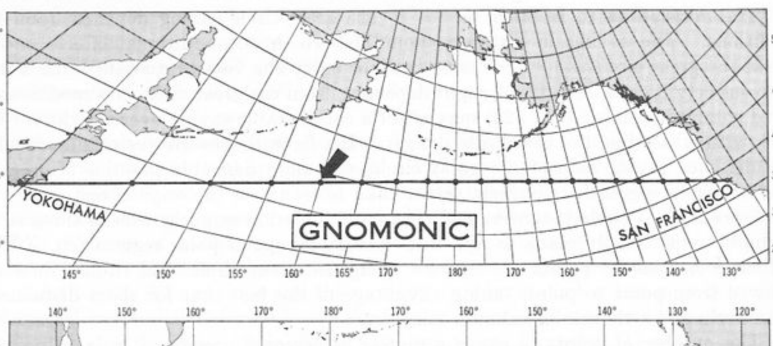

FIGURE 820.—Transferring great-circle points from a gnomonic chart to a Mercator chart.

A projection on which a straight line is *approximately* a great circle can be used in place of a gnomonic chart with negligible error. If such a projection is conformal, as in the case of the Lambert conformal (art. 314), measurement of course and distance of each leg can be made directly on the chart, as explained in article 2511.

Some great circles are shown on pilot charts and certain other charts, together with the great-circle distances. Where tracks are recommended on charts or in sailing directions, it is good practice to follow such recommendations.

821. Great-circle sailing by conversion angle.—The direction of the great circle at the point of departure is called the **initial great-circle course,** and its direction at the destination is called the **final great-circle course.** The *difference* between the initial great-circle course and the single rhumb line course is called **conversion angle.** This is usually about half the difference between initial and final great-circle courses.

Conversion angles for difference of longitude to 120°, sufficient for virtually all situations in which great-circle sailing is likely to be used by ships, are given in table 1. To use the table, measure the rhumb line course on a Mercator chart (or compute it by Mercator or mid-latitude sailing) and apply the conversion angle to find the initial great-circle course. The sign of the correction can be determined by means of the tabulation at the bottom of the table. With a little practice, one can determine the sign mentally by remembering that the great-circle course always lies nearer the pole (in the hemisphere of the point of departure) than the rhumb line course, except for those values given in italics.

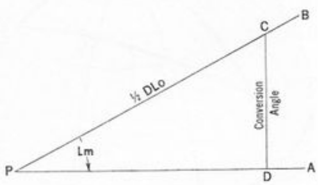

FIGURE 821.—Graphical solution for conversion angle.

The use of the conversion angle as taken directly from the table results in a course line tangent to the great circle (as plotted on a Mercator chart) and hence one that carries the vessel to higher latitudes than the great circle. To convert this to the corresponding chord, as in great-circle sailing by chart, divide the conversion angle by the number of legs, and *subtract* this value from the tabulated conversion angle before applying the correction to the rhumb line course. At the end of each leg make a new solution, using the position of the vessel as the point of departure.

This method does not indicate the suitability of the route unless the entire solution is made in advance and the results plotted on a chart.

Approximate values of conversion angle can be found by the formula:

$$\tan \text{conversion angle} = \frac{\sin \text{Lm} \tan \frac{1}{2}\text{DLo}}{\cos \frac{1}{2} l}$$

if both points are on the same side of the equator. For small differences of latitude, $\cos \frac{1}{2} l$ can be considered 1 without introducing a significant error. The tangent of a small angle equals, approximately, the angle itself (in radians). Therefore, for small values of DLo and l (up to 15° to 20°) the formula can be simplified:

$$\text{conversion angle} = \tfrac{1}{2}\text{DLo} \sin \text{Lm}.$$

This formula can be solved graphically (fig. 821). Draw any line *PA*, and from *P* draw *PB* making an angle with *PA* equal to Lm. Along *PB* measure ½ DLo, letting any convenient linear unit equal 1°. From *C*, the point thus found, draw *CD* perpendicular to *PA*. The length of *CD* in the units used for ½ DLo is the conversion angle in degrees. Conversion angle can also be determined by table 3, using mid latitude as course, ½ DLo as *D*, and conversion angle as p. The value found by formula, however solved, may not be accurate for large differences of latitude.

822. Great-circle sailing by computation.—In figure 822, 1 is the point of departure, 2 the destination, P the pole nearer 1, $1XV2$ the great circle through 1 and 2, V the vertex, and X any point on the great circle. The arcs $P1$, PX, PV, and $P2$ are the colatitudes of points 1, X, V, and 2, respectively. If 1 and 2 are on opposite sides of the equator, $P2$ is $90° + L_2$. The length of arc 1–2 is the great-circle distance between 1 and 2. Arcs 1–2, $P1$, and $P2$ form a spherical triangle. The angle at 1 is the initial great-circle course from 1 to 2, that at 2 the supplement of the final great-circle course (or the initial course from 2 to 1), and that at P the DLo between 1 and 2.

Great-circle sailing by computation usually involves solution for the initial great-circle course; the distance; latitude and longitude, and sometimes the distance, of the vertex; and the latitude and longitude of various points (X) on the great circle. The computation for initial course and the distance involves solution of an oblique spherical triangle, and any method of solving such a triangle can be used. If 2 is the **geographical position (GP)** of a celestial body (the point at which the body is in the zenith), this triangle is solved in celestial navigation, except that $90° - D$ (the altitude) is desired instead of D. The solution for the vertex and any point X usually involves the solution of right spherical triangles.

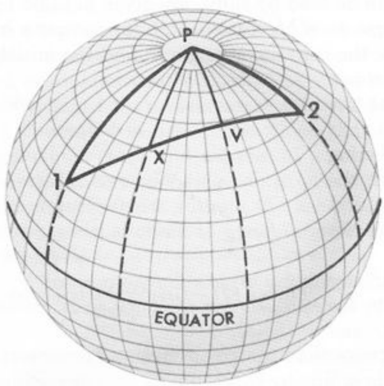

FIGURE 822.—The navigational triangle of great-circle sailing.

Although various formulas can be used, haversine formulas are considered most suitable for determining initial course and the distance, as these avoid the ambiguity that may arise through the use of trigonometric functions which do not indicate the quadrant in which the answer lies. In the formulas given below, the subscripts refer to the points indicated in figure 822. All terms without subscripts are from 1 to 2, D_v and DLo_v are from 1 to V, and D_{vx} and DLo_{vx} are from V to X. Other quantities can be computed by interchanging 1 and 2 in figure 822 and using the same formulas. The following formulas are suitable for great-circle sailing by computation:

$$\text{hav } D = \text{hav DLo} \cos L_1 \cos L_2 + \text{hav } l$$

which may be written $\text{hav } D = \text{hav } \theta + \text{hav } l$ (where $\text{hav } \theta = \text{hav DLo} \cos L_1 \cos L_2$)

$$\text{hav } C = \sec L_1 \csc D\, [\text{hav coL}_2 - \text{hav } (D \sim \text{coL}_1)]$$

$$\cos L_v = \cos L_1 \sin C$$

$$\sin \text{DLo}_v = \cos C \csc L_v$$

$$\sin D_v = \cos L_1 \sin \text{DLo}_v$$

$$\tan L_x = \cos \text{DLo}_{vx} \tan L_v$$

Example.—A ship is proceeding from Manila to Los Angeles. The captain wishes to use great-circle sailing from lat. 12°45′.2 N, long. 124°20′.1 E, off the entrance to San Bernardino Strait, to lat. 33°48′.8 N, long. 120°07′.1 W, five miles south of Santa Rosa Island.

Required.—(1) The initial great-circle course.

(2) The great-circle distance.